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Distribution of Open Intervals*

THE distribution of open birth interval has attracted many authors ever since it was suggested by Srinivasan[4] in 1966 as an index of fertility. He defines the j-th open interval, U_j fora woman in the j-th parity on the date of the survey as the interval between thej-th live-birth and the survey date. Srinivasan has shown that the open interval, is more sensitive to changes in fertility than the closed interval. Further the open intervals are much less contaminated by recall lapses, interview bias, etc. than the closed interval. Srinivasan has also derived the moments of open interval in terms of the moments of the closed interval, Tj. Leridon[2] treats the open birth interval as a backward recurrence time and provides some improvements over Srinivasan's expressions. Venkatacharya[5] has also done some empirical studies on open intervals. Sheps and others[3] have studied the truncation effects on open and closed intervals.

We have attempted here to carry forward these series of writings by focussihg attention on the distribution of open intervals in relation to that of the closed intervals. Further, we seek to obtain also a method to predict the distribution of open interval for a future date based on the data for a specified date. The distribution of the closed intervals obtained by George[1] has been used for simulating the distribution of open intervals for a future date.

Notations

Let us explain the notations that will be used in this paper

*Abstract published in the Proceedings of the Indian Science CangresS, 1972.

T_j ($j \geqslant 1$) = interval between the j-th and ($j+1$)-th live births. (2.1)
T_0=Interval between the date of consummation of the marriage and the first birth. (2.2)

(t)=distribution function of T_j (over women) given that

$$T_j < \infty = \text{prob.} \{T_j \leqslant y/T_j < \infty\}. \quad (2.3)$$

U_{js} ($j \geqslant 1$)=interval between the birth of the j-th child and the survey date, γ_s ($s = 0 \;\; 1$) for women of parity j at the time of the survey. (2.4)
U_{0s}=Interval between the date of the consummation of marriage and the survey date γ_s, ($s = 0, 1$) for women of parity 0 at the time of the survey. (2.5)

$\alpha_j (t, x)$=probability that a woman, who has entered the state j (j-th parity) at age x and has spent t units of time in the state j will ever proceed to ($j+1$)-th state (parity progression ratio, *PPR*, at time t). (2.6)

$f_j (x)$=probability density of x, the age at which the women enter the state j. (2.7)

$P_{ij}^{(t_1)} (t)$ = probability that a woman who has spent t units of time in state i before γ_0 is in state j at γ_1 (t_1 = time between γ_0 and γ_1). (2.8)

$P_j (t)$=probability that a woman who has already spent t units of time in state j will never leave the state j. (2·9)

P_j=P_j (0)=probability that a woman entering the j-th state will never leave the state j. (2.10)

$S_j (t, x)$=conditional probability that a woman entering the j-th state at age x will survive for at least t units of time given that she will never leave state j. (2.11)

$\beta_j (t_1, t, x)$=the probability that a woman who entered the state j at age x and has already spent t units of time in state j will survive for another t_1 units of time

$$= \frac{S_j (t + t_1, x)}{S_j(t, x)} \quad (2.12)$$

$(t, t_1) = \int \beta_j (t, t_1, x) f_j (x)\, dx$ = probability that a woman who has spent t units of time in the j-th state will survive for t_1 more units of time. (2.13)

$q_j (x)$=probability that a woman entering the j-th state at age x will never leave the state j. (2.14)

$[S_j (t, x)\, q_j (x)]$=probability that a woman entering the j-th state at age x will never leave state j and will survive for at least t units of time. (2.15)

$n_{js} (t)$=number of women of j-th parity at time γ_s (s=0, 1) having open interval U_{js} between t and $t+\delta$, where δ is the width of the class interval. (2.16)

3. Distribution of Open Intervals

The distribution of open intervals is obtained under the assumption that the number of women entering the state j on each day is the same, i.e. the j-th live-births are distributed uniformly over time. A woman who gave birth to the j-th live child t units of time before the survey date γ_0 should belong to one of the two groups : (*i*) those who will never leave the state j and (*ii*) those who will ever leave state j, with probabilities $q_j (x)$ given in (2.14) and $(1 - q_j (x))$ respectively where x denotes the age at which she enters the state j. Probability that a woman will be in the j-th state at the time of the survey γ_0, given that she belongs to the second group is given by $1-F_j (t)$, where $F_j (t)$ is given in (2.3). Probability that a woman is in state j at time γ_0, given that she belongs to the first group, is the probability that she survives until the survey date γ_0 and is denoted by $s_j (t, x)$ where x denotes the age at which she enters the state j. The probability that a woman, who entered j-th state, t units of time before γ_0 is still in the same state j at γ_0 is given by

$$P\{t \leqslant U_{jo} < t + dt\}$$

$$= \frac{\int\{(1 - q_j (x)) (1 - F_j (t)) + q_j (x)\, S_j (t, x)\} f_j (x)\, dx}{\int[\int\{(1 - q_j (x)) (1 - F_j (t)) + q_j (x)\, S_j (t, x)\} f_j (x)\, dx]\, dt}\, dt; \quad (3.1)$$

where $f_j (x)$ is as given in (2.7). The expression in (3.1) can be used only if the information on age specific parity progression ratio and also data on age at which women enter the j-th state, are available. When these are not available use the expressions of $S_j (t, x)$, $q_j (x)$ and $[S_j (t, x)\, q_j (x)]$

given in (2.11), (2.14) and (2.15) respectively, with respect to age to obtain the probability density of U_j. The expression of $[S_j(t, x)\, q_j(x)]$ with respect to x is the probability that a woman in the j-th state will never leave state j and survive for at least t units of time. It can be obtained as the probability that a woman in state j will never leave state j multiplied by the conditional probability that she will survive for at least t units of time given she will never leave state j, which is equal to $E[q_j(x)]\, E[S_j(t, x)]$.

Let, $$q_j = E[q_j(x)] = \int q_j(x) f_j(x)\, dx, \tag{3.2}$$

and $$S_j(t) = E(S_j(t, x)] = \int_j (t, x) f S_i(x)\, dx. \tag{3.3}$$

Hence the expression in (3.1) can be modified using (3.2) and (3.3) as

$$p\{t \leqslant U_j < t + dt\}$$

$$= \frac{(1 - q_j)(1 - F_j(t)) + q_j S_i(t)}{\int[(1 - q_j)(1 - F_j(t)) + q_j S_j(t)]\, dt}\, dt, \tag{3.4}$$

where, $$\int[S_j(t, x)\, q_j(x)] f_j(x)\, dx = E[S_j(t, x)\, q_j(x)] = S_j(t)\, q_j. \tag{3.5}$$

The expressions in (3.1) and (3.5) give the probability that the open interval, U_j, will lie between t and $t + dt$.

4. Simulation of the Distribution of the Open Birth Intervals

The problem is to predict the frequency distribution of open birth intervals, U_{j1} for parity j at a future date γ_1 if the distribution of open birth intervals $U_{j-1,0}$ and U_{j0} for parities $(j - 1)$ and j at γ_0 is available as the grouped data with class intervals of width δ. Let t_1 be the time between γ_0 and γ_1 which is assumed to be an integral multiple of δ. Further it is assumed that no woman can have more than one live-birth within t_1 units of time. This assumption may be given up ; but then the formulae will have to be modified so as to accommodate those women reaching parity j, from earlier parities, $j - 2$, $i - 3$ etc., within t_1 units of time.

Let us consider only eligible women. A woman is said to be in the j-th state if she has given birth to exactly j $(j = 1, 2, \ldots)$ live children and U_{j0} denotes the time that she has spent in the j-th state before the observation is made and T_j is the total time spent by a woman in the j-th state. $1 - F_j(t)$ is the probability that a woman will spend at least t units of time in the j-th state given that she will ever leave the state.

$n_{j0}(t)$, the number of women in the j-th state having U_{j0} between t and $t+\delta$ at γ_0, consists of two groups, viz. (*i*) $n_{j0}^{(1)}(t)$ women, who have an open interval, U_{j0}, between t and $t+\delta$ and will never leave the state j and (*ii*) $n_{j0}^{(2)}(t)$ women who have an open interval U_{j0} between t and $t+\delta$ who will eventually leave the state j. To estimate $n_{j0}^{(1)}(t)$ and $n_{j0}^{(2)}(t)$, the expression for $P_j(t)$, the probability that a woman who has already spent t units of time in state j will never leave the state j, has to be first derived.

$\alpha_j(t, x)$ given in (2.6) is the conditional probability that a women who has spent t units of time in state j will ever leave the state given that she has entered the state j at age x. Hence from (2.6) and (2.7),

$\int \alpha_j(t, x)\, f_j(x)\, dx$ = probability that the woman will ever leave the state j given that she has reached the state j, and (4.1)

$$P_j(t) = 1 - \int \alpha_j(t, x) f_j(x)\, dx$$
$$= P\{T_j = \infty \mid T_j \geqslant t\}. \tag{4.2}$$

$U_{j0} = t$ is a random sample from $T_j \geqslant t$ since the survey date γ_0 is a random point. But $P_j(t)$, the probability that a woman with an open interval U_{j0} equal to t will not leave the state j, is given by the conditional probability that T_j is infinite given $T_j \geqslant$ t.

Hence, $$P\{T_j = \infty \mid U_{j0} = t\}$$

$$= P\{T_j = \infty \mid T_j \geqslant t\} = \frac{P\{T_j = \infty,\ T_j \geqslant t\}}{P\{T_j \geqslant t\}},$$

$$= \frac{P(T_j = \infty)}{P(T_j \geqslant t)} = \frac{P_j(0)}{P_j(0) + [1 - P_j(0)]\,[(1 - F_j(t)]},$$

$$= P_j(t), \tag{4.3}$$

where, $P\ (T_j \geqslant t) = P_j\,(0) + [1 - P_j\,(0)]\,[1 - F_j\,(t)].$ (4.4)

Hence,

$$n_{j0}^{(1)}\ (t) = n_{j0}\ (t)\ P_j\ (t), \tag{4.5}$$

and

$$n_{j0}^{(2)}\ (t) = n_{j0}\ (t)\ [1 - P_j\ (t)], \tag{4.6}$$

where, $P_j\,(t)$ is given either by (4.2) or by (4.3). $P_j\,(0)$ does not occur in (4.2) whereas it is involved in (4.3). All the $n_{j0}^{(1)}\ (t)$ women of first group who survive for t_1 units of time will remain in state j until γ_1. The proportion of women in second group remaining in the same state j at γ_1 is given by the conditional probability of a woman in state j at γ_0 remaining in the same state j until γ_1 given that $U_{j0} = t$ at γ_0. Hence

$$P_{jj}^{(t_1)}\ (t) = P\ \{U_{j1} = \overline{t + t_1} \mid U_{j0} = t\},$$

$$= P\ \{ T_j \geqslant \overline{t + t_1} \mid T_j \geqslant t\},$$

$$= \frac{P\ \{T_j \geqslant t,\ T_j \geqslant \overline{t + t_1}\}}{P\ \{T_j \geqslant t\}} = \frac{P\ \{T_j \geqslant t + t_1\}}{P\ \{\ T_j \geqslant t\}},$$

$$= \frac{1 - F_j\ (t + t_1)}{1 - F_j\ (t)}, \tag{4.7}$$

where $T_j < \infty$. In the remaining portion of this section, wherever T_j occurs, it implies that $T_j < \infty$.

The women in the j-th state having U_{j1} between $(t + t_1)$ and $(t + t_1 + \delta)$ are those among the $n_{j0}\ (t)$ women who do not leave state j in t_1 units of time between γ_0 and γ_1, and their number is given by

$$n_{j1}\ (t + t_1) = \beta_j\ (t,\ t_1)\ n_{j0}^{(1)}\ (t) + n_{j0}^{(2)}\ (t)\ \ P_{jj}^{(t_1)}\ (t), \tag{4.8}$$

where, $P_{jj}^{(t_1)}\ (t)$ is given by (4.7).

The expression in (4.8) can be applied to the data on U_{j0} to obtain the frequencies for U_{j1} for class intervals with the lower class limit at least as large as t_1.

The women having open interval U_{j1} between $t_1 - i\delta$ and $t_1 - \overline{i-1}\delta$ at γ_1 are those who are in the $(j-1)$-th state at γ_0 and do not change to state j in $(i-1)\delta$ units of time after γ_0 but move to state j before $i\delta$ units of time. The number of women belonging to this group is given by

$$n_{j1}(t_1 - i\delta) = \sum_t n^{(2)}_{j-1,0}\ (t)\, P^{(\overline{i-1}\delta)}_{j-1,\ j-1}(t)\, P^{(\delta)}_{j-1,\ j}{}^{(t+\overline{i-1}\delta)}, \tag{4.9}$$

where,
$$P^{(\overline{i-1}\delta)}_{j-1,\ j-1}(t) = P\{U_{j-1} = t + \overline{i-1}\,\delta \mid U_{j-1,\,0} = t\}$$

$$= P\{T_{j-1} \geqslant t + \overline{i-1}\,\delta \mid T_{j-1} \geqslant t\}$$

$$= \frac{P\{T_{j-1} \geqslant t + \overline{i-1}\,\delta,\ T_{j-1} \geqslant t\}}{P\{T_{j-1} \geqslant t\}}$$

$$= \frac{1 - F_{j-1}(t + \overline{i-1}\,\delta)}{1 - F_{j-1}(t)}\ , \tag{4.10}$$

$$P^{(\delta)}_{j-1,\ j}\ \ (t + (\overline{i-1}\delta)) = P\{T_{j-1} \leqslant t + i\,\delta \mid U_{j-1} = t + \overline{i-1}\,\delta\},$$

$$= P\{T_{j-1} \leqslant t + i\,\delta \mid T_{j-1} \geqslant t + \overline{i-1}\,\delta\}$$

$$= \frac{P\{T_{j-1} \leqslant t + i\,\delta\ ;\ T_{j-1} \geqslant t + i - 1\,\delta\}}{P\{T_{j-1} \geqslant t + \overline{i-1}\delta\}}$$

$$= \frac{F_{j-1}(t + i\,\delta) - F_{j-1}(t + \overline{i-1}\delta}{1 - F_{j-1}(t + i-1\delta)}, \tag{4.11}$$

and

$$P^{(\overline{i-1}\delta)}_{j-1;\ j-1}(t)\ P^{(\delta)}_{j-1,\ j}\ (t + \overline{i-1}\,\delta) = \frac{F_{j-1}(t + i\,\delta) - F_{j-1}(t + \overline{i-1\,\delta})}{1 - F_{j-1}(t)}\ ,$$

$$= \frac{P\{t + \overline{i-1}\,\delta \leqslant T_{j-1} \leqslant t + i\,\delta\}}{P\{T_{j-1} \geqslant t\}}. \tag{4.12}$$

Expression in (4.11) is the conditional probability of a transition from the $(j-1)$-th state to the j-th state not within $(i-1)\delta$ units of time but within $i\delta$ units of time after γ_0 given that a time t has already been spent in $(j-1)$-th state before γ_0. Hence expression in (4.9) reduces to

$$n_{j1}(t_1 - i\delta) = \sum_t n^{(2)}_{j-1,0}(t) \frac{P\{t + \overline{i-1}\,\delta \leqslant T_{j-1} < t + i\delta\}}{P\{T_{j-1} \geqslant t\}},$$

$$= \sum_t n^{(2)}_{j-1,0}(t) \frac{[F_j(t + i\delta) - F_j(t + \overline{i-1\,\delta})]}{[1 - F_j(t)]}. \quad (4.13)$$

Expression in (4.9) can be applied to data on $U_{j-1,0}$ at Υ_0 to predict the frequencies in class intervals with the lower limit less than t_1.

5. Estimation of P_j

To estimate P_j, which is given in (2.10), a method using the data on completed fertility is discussed in this section. Let us consider a group of N married women, who have completed their fertility and observe the state to which each one of them belongs. Let n_i be the number of women who have completed their fertility while in the i-th state $(i = 0, 1, 2, \ldots)$. For a woman to be in the i-th state it is necessary that she must have passed through the state 0, 1, 2, . . . and $i - 1$ before reaching the state i.

The number of women who ever arrived in state i is given by $\sum_{k \geqslant i} n_k$. The probability of a woman in state i ever reaching state j $(j > i)$ may then be estimated by

$$P_{ij} = \frac{\sum_{k \geqslant j} n_k}{\sum_{k \geqslant i} n_k}. \quad (5.1)$$

The probability that a woman in state j will never leave state j is one minus the probability that she will ever leave the state j which is given by

$$P_j = 1 - P_{j,\,j+1}. \quad (5.2)$$

Let P'_{ij} be the probability that a woman who reached state i will reach state j and then never leave state j. P'_{ij}'s may be estimated from

$$P'_{ij} = \frac{n_j}{\sum_{k \geqslant i} n_k} \quad (5.3)$$

P'_{ij}'s estimated for 1701 women married during 1920-1939, taken from Population Register maintained by the Department of Statistics, University of Kerala, are given in Table 1.

TABLE 1--ESTIMATES OF P'_{ij} ($j \geq i$)

Parity	0	*1*	*2*	*3*	*4*	5	*6*	*7*	*8*	9	*10*	*11*	*12+*
0	0.0165	0.0259	0.0488	0.0541	0.0882	0.1082	0.1240	0.1422	0.1399	0.1088	0.0717	0.0435	0.0282
1	0	0.0263	0.0496	0.0550	0.0897	OI1CO	0.1261	0.1447	0.1423	0.1105	0.0729	0.0442	0.0287
2	0	0	0.0509	0.0565	0.0921	0.1130	0.1295	0.1486	0.1461	0.1136	0.0749	0.0454	0.0294
3	0	0	0	0.0595	0.0970	0.1190	0.1365	0.1565	0.1538	0.1199	0.0789	0.0479	0.0310
4	0	0	0	0	0.1032	0.1265	0.1451	0.1664	0.1637	0.1272	0.0840	0.0509	0.0330
5	0	0	0	0	0	0.1411	0.1618	0.1856	0.1825	0.1419	0.0936	0.0567	0.0368
6	0	0	0	0	0	0	0.1884	0.2161	0.2125	0.1652	0.1089	00660	0.0429
7	0	0	0	0	0	0	0	0.2662	0.2618	0.2035	0.1343	0.0814	0.0528
8	0	0	0	0	0	0	0	0	0.3568	0.2774	0.1829	0.1109	0.0720
9	0	0	0	0	0	0	0	0	0	0.4312	0.2844	0.1725	0.1119
10	0	0	0	0	0	0	0	0	0	0	0.5000	0.3032	0.1968
11	0	0	0	0	0	0	0	0	0	0	0	0.6066	0.3934
12+	0	0	0	0	0	0	0	0	0	0	0	0	1

The first row of Table 1 provides the absolute probabilities that a married woman will have no live-birth, one live-birth and so on. The second row corresponding to parity one gives the conditional probabilities that a woman will stop at first child, second child etc., given that she already had a first child. The first element in this row is zero, because the conditional probability that a woman will stop at the zero state given that she already had a child, is zero. For the same reason $P'_{ij} = 0$, when $j < i$.,

P_{ij}, $(j > i)$, the probability of ever reaching state j given that the woman is in state i can be obtained by adding the elements of the i-th row starting from the j-th element. Thus the probability that a woman in state 3, having already 3 live-births, will ever reach state 5, is equal to sum of all the elements in the third row starting from the fifth element. The required probability is given by

$$P_{3,5} = 0.8435. \tag{5.4}$$

The probability of never leaving state j is given by the diagonal element in the j-th row. The estimates provided in the row corresponding to state zero depends on 1701 women in the sample ; row corresponding to state one depends on 1673 women who have ever reached the state one and so on.

Table 1 is also useful for determining the expected number of births prevented by one sterilisation. Let the sterilisation be done only if the woman had k live births. The expected number, E_k of live births prevented by sterilising a woman who had exactly k live births is given by

$$E_k = \sum_{j \geqslant k} (j - k) P'_{kj} \ . \tag{5.5}$$

If $k = 3$, the expected number is obtained from Table 1 as

$$E_3 = \sum_{j \geqslant k} (j - 3) P'_{3j} = 4.0734. \tag{5.6}$$

E_3 is the expected number of live-births prevented by effectively sterilising a woman who already had 3 live children or it is the expected number of additional children she would have had in her life time if she were not effectively sterilised. Let $\{g_i\}$ be the distribution of parity, i, among sterilised women when women of parity k or more are sterilised. Then the number of births averted by one sterilisation is $\sum_{i \geqslant k} E_i g_i$.

6. An Example for the Method of Simulation

In this section the method discussed in Section 4 is applied to distribution of open intervals, as on August 17, 1968, (γ_0) collected for the Kerala Standard Fertility Survey in the Department of Statistics, University of Kerala. The distributions of second and third parity open intervals have been used to predict the distribution of third parity open intervals for August 17, 1969 (γ_1). The actual observed distribution of third parity open intervals on this date is also provided for comparison.

The completed intervals are assumed to follow a distribution as in George[1]. The parameters are estimated by the method of moments from the data on closed intervals obtained from the same survey. The estimates are given by, $k = 20$, $a = 0.12487$, $b = 0.07679$ and $c = 0.04878$ for the second parity and $k = 20$, $a = 0.12706$, $b = 0.07448$ and $c = 0.05297$ for the third parity. Using these estimates the actual theoretical frequencies are calculated for the closed intervals for the above model. $P_2(0)$ and $P_3(0)$ are obtained from Table 1 as 0.0509 and 0.0595 respectively. The observed frequencies $n_{20}(t)$ and $n_{30}(t)$ for parities two and three and the estimated frequencies $n_{20}^{(1)}(t)$, $n_{20}^{(2)}(t)$, $n_{30}^{(1)}(t)$ and $n_{30}^{(2)}(t)$ for the groups under each parity, provided in Table 2, are used for simulating the distribution of third parity open intervals.

Columns (2) and (5) of Table 2 represent observed frequencies $n_{20}(t)$ and $n_{30}(t)$ of open intervals. Columns (3) and (6) are obtained by applying expression (4.5) to columns (2) and (5) respectively. Column (4) can be obtained either using (4.6) or by subtracting column (3) from column (2). Similarly Column (7) can be obtained by subtracting Column (6) from Column (5).

Using (4.8) and (4.9) and the estimated frequencies in Table 2, the distribution of third parity open intervals has been simulated for August 17, 1969 and given in Table 3. Expression (4.8) involves $\beta_j(t, t_1)$ the probability that a woman who has already spent 't' units of time in the j-th parity and does not go for another live birth, survives for 't_1' units of time and this has been assumed to be equal to 1. Since for the women in the age group 15-45 the survival rate is rather very high and the women considered in this instance are only those who do not go to the next parity (no risk associated with labour) the above assumption, though not accurate, is justified.

TABLE 2—OBSERVED FREQUENCIES FOR PARITIES 2 AND 3 ON AUGUST 17, 1968 fr$_c$) AND ESTIMATED FREQUENCIES FOR THE TWO GROUPS UNDER EACH PARITY

t	Parity 2			Parity 3		
	$n_{20}(t)$	$n_{20}^{(1)}(t)$	$n_{20}^{(2)}(t)$	$n_{30}(t)$	$n_{30}^{(1)}(t)$	$n_{30}^{(2)}(t)$
1	(2)	(3)	(4)	(5)	(6)	(7)
0-9	54	2.75	51.25	39	2.32	36.68
9-12	11	0.57	10.43	14	0.84	13.16
12-15	9	0.48	8.52	11	0.68	10.32
15-18	15	0.83	14.17	16	1.04	14.96
18-21	11	0.65	10.35	16	1.11	14.89
21-24	19	1.22	17.78	13	0.99	12.01
24-27	13	0.92	12.08	6	0.51	5.49
27-30	9	0.73	8.27	15	1.46	13.54
30-33	6	0.55	5.45	7	0.79	6.21
33-36	9	0.95	8.05	6	0.77	5.23
36-39	5	0.60	4.40	12	1.77	10.23
39-42	7	0.95	6.05	5	0.84	4.16
42-45	5	0.77	4.23	11	2.11	8.89
45-48	6	1.02	4.98	7	1.53	5.47
48-51	3	0.58	2.42	5	1.23	3.77
51-54	3	0.65	2.35	2	0.55	1.45
54-57	3	0.72	2.28	9	2.78	6.22
57-60	2	0.54	1.46	2	0.69	1.31
60+	54	16.22	37.78	59	22.43	36.57

It can be seen from Table 3 that the observed and expected frequencies do not differ much except at the two tails. In the class interval 0-9 the predicted frequency is much higher than the observed frequency and in the class 60 and over the observed frequency is much higher. This discrepancy may be due to the fact that the parity progression ratios were estimated on the basis of data on completed fertility, collected from a cohort of women married during the period 1920-1939. The instantaneous parity progression ratios during 1968-1969 may be much smaller than for the above cohort, i.e. we have actually worked with parity progression ratios higher than the actual ones. A higher parity progression ratio *(PPR)* for parity 2 has inflated the expected frequencies in class intervals 0-9 and 9-12, whereas a higher *PPR* for parity 3 has deflated the expected frequency in the class interval 60 and over. Since data on instantaneous *PPR* were not available the aforesaid estimates had to be used for illustration of the method.

TABLE 3—DISTRIBUTION OF OPEN INTERVAL *U*, FOR PARITY 3, ON AUGUST 17, 1969 (YI)

U	*Projected frequency*	F_p†	*Observed frequency*	F_0†
0-9	46.89	0.1900	37	0.1365
9-12	16.22	0.2557	14	0.1882
12-15	12.88	0.3079	20	0.2620
15-18	12.51	0.3586	13	0.3100
18-21	11.90	0.4067	12	0.3542
21-24	11.89	0.4550	11	0.3948
24-27	8.72	0.4903	7	0.4201
27-30	11.66	0.5375	12	0.4649
30-33	10.79	0.5813	8	0.4944
33-36	8.08	0.6140	11	0.5351
36-39	3.58	0.6285	3	0.5461
39-42	8.77	0.6640	10	0.5830
42-45	4.08	0.6806	5	0.6018
45-48	3.54	0.6949	4	0.6162
48-51	7.19	0.7240	7	0.6421
51-54	3.04	0.7364	3	0.6531
54-57	6.82	0.7640	11	0.6937
57-60	4.43	0.7819	7	0.7196
60+	53.82	1.0000	76	1.0000
Mean	36.35		41.83	
Standard Deviation	29.53		31.00	

†F_p and F_0 are the cumulative distribution functions based on the projected and observed frequencies respectively.

Migration, which has not been considered in the model, is another factor that might have affected the observed frequencies. There are only 39 women in the class interval 0-9 on 17th August 1968, but in the corresponding classes (12-21) on 17th August 1969 there are 45 women observed. This increase can be attributed only to migration.

The projected and observed distributions were converted into probability distribution functions, F_P and F_0 respectively. Treating the projected distribution function as the theoretical one, the Kolmogorov-Smirnov-one sample statistic, is used, to test for difference between the two distributions. In spite of the factors discussed in the preceding two paragraphs, the two distributions are not significantly different at 5% level $\left(\max |F_p - F_0| = 0.0869 < \frac{1.52}{\sqrt{271}} = 0.0923\right)$. It may also be noted that if the differences at the extremes were not so large, $\max |F_p - F_0|$ would have been much smaller and not significant even at 1% level. (1% critical value is $\frac{1.36}{\sqrt{271}} = 0.0826$).

The mean and standard deviation of the predicted distribution are 36.35 and 29.53 respectively while that of the observed distribution are 41.83 and 31.00. These values are not statistically different. The mean of the predicted distribution has been deflated by the higher value of *PPR* as discussed earlier. The curves for the two distribution functions have nearly the same form. The curves are horizontal upto nine months and then decrease steadily, but never reaching zero, since there is a nonzero probability that a woman does not go for another live birth. This suggests that there is a non-zero probability for an infinite open interval. So to make the distribution of open intervals a 'proper distribution', the interval will have to be truncated. In reality also this is needed, in the sense that when a woman completes her fertility period, the open interval associated may be truncated. This may vary from woman to woman. We will have to accept some standard value, say 120 months, for truncating the distribution.

7. Conclusion

The distribution of open intervals has been studied in relation to that of the closed intervals. This has been made use of, to simulate the distribution for a future date. The simulation process makes use of parity progression ratios. In the example given in this paper, the *PPR* was estimated for a cohort of women married during 1920-1940. The effect of a higher value of the *PPR* is to shift the distribution slightly to the left, In spite of this, the method of simulation is found to give good results.

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